Nonequilibrium phase transition in the kinetic Ising model: Existence of a tricritical point and stochastic resonance

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The dynamic phase transition has been studied in the two-dimensional kinetic Ising model in the presence of a time varying (sinusoidal) magnetic field by Monte Carlo simulation. The nature (continuous or discontinuous) of the transition is characterized by studying the distribution of the order parameter and the temperature variation of the fourth-order cumulant. For the higher values of the field amplitude the transition observed is discontinuous and for lower values of the field amplitude it is continuous, indicating the existence of a *tricritical point* (separating the nature of transition) on the phase boundary. The transition is observed to be a manifestation of *stochastic resonance*. [S1063-651X(99)07801-0]

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I. INTRODUCTION

The kinetic Ising model in the presence of an oscillating magnetic field gives rise to various interesting dynamical responses [1]. The dynamic phase transition, hysteresis [2], and *stochastic resonance* [3] are the most important dynamic responses of recent interest. Tome and de Oliveira [4] observed and studied the dynamic transition in the kinetic Ising model in the presence of a sinusoidally oscillating magnetic field. They solved the mean field dynamic equation of motion (for the average magnetization) of the kinetic Ising model in the presence of a sinusoidally oscillating magnetic field. By defining the order parameter as the time averaged magnetization over a full cycle of the oscillating magnetic field they showed that the order parameter vanishes depending upon the value of the temperature and the amplitude of the oscillating field. In the field amplitude and temperature plane they have drawn a phase boundary separating dynamic ordered (nonzero value of the order parameter) and disordered (the order parameter vanishes) phases. They [4] have also predicted a *tricritical point* (TCP), separating the nature (discontinuous/continuous) of the transition on the phase boundary line. However, such a transition, observed [4] from the solution of the mean field dynamical equation, is not truly dynamic. This is because, for the field amplitude less than the coercive field (at temperature less than the transition temperature without any field), the response magnetization varies periodically but asymmetrically even in the zero frequency limit; the system remains locked to one well of the free energy and cannot go to the other one, in the absence of noise or fluctuations.

Lo and Pelcovits [5] attempted to study the dynamic nature of this phase transition (incorporating the effect of fluctuations) in the kinetic Ising model by Monte Carlo (MC) simulation. In this case, the transition disappears in the zero-frequency limit; due to the presence of fluctuations, the magnetization flips to the direction of the magnetic field and the dynamic order parameter vanishes. However, they [5] have not reported any precise phase boundary. Acharyya and Chakrabarti [2] studied the nonequilibrium dynamic phase transition in the kinetic Ising model in the presence of an oscillating magnetic field by an extensive MC simulation. They [2] have successfully drawn the phase boundary for the dynamic transition and predicted a tricritical point on it. It was also noticed by them [2] that this dynamic phase transition is associated with the breaking of the symmetry of the dynamic hysteresis loop. In the dynamically disordered (the value of order parameter vanishes) phase the corresponding hysteresis loop is symmetric and in the ordered phase it loses its symmetry (giving a nonzero value of the dynamic order parameter).

Recent studies that reveal the thermodynamic nature of the dynamic transition are on the temperature variations of ac susceptibility [2], the relaxation behavior of the dynamic order parameter and the divergence of the time scale (critical slowing down) [6], the scaling of the distribution of dynamic order parameter and the divergence of the length scale [7], and on the temperature variation of the dynamic correlation [8].

Although the existence of a TCP has been predicted from the temperature variations of the average order parameter [2,4], a detailed and systematic study has not yet been performed to detect the nature (continuous/discontinuous) of the dynamic transition along the dynamic phase boundary. In this paper the statistical distribution of the dynamic order parameter has been studied to detect the nature of the transition, by Monte Carlo simulation in a two-dimensional kinetic Ising model in the presence of an oscillating magnetic field. The temperature variation of the fourth-order cumulant [9] (of the distribution of the dynamic order parameter) has also been studied to characterize the transition. The relation between the stochastic resonance [3] and dynamic transition [2] is also discussed. The paper is organized as follows. In Sec. II the model and the MC simulation scheme are discussed. Section III contains the simulational results. The paper ends with a summary of the work in Sec. IV.

II. DESCRIPTION OF THE MODEL AND THE SIMULATION SCHEME

The Hamiltonian of an Ising model (with a ferromagnetic nearest-neighbor interaction) in the presence of a time vary-

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ing magnetic field can be written as

$$H = -J\sum_{\langle ij\rangle} s_i s_j - h(t)\sum_i s_i. \quad (2.1)$$

Here $s_i\ (=\pm 1)$ is the Ising spin variable, $J>0$ is the ferromagnetic spin-spin interaction strength, and $h(t)$ is the sinusoidally oscillating (in time but uniform in space) magnetic field. The time variation of $h(t)$ can be expressed as

$$h(t) = h_0 \cos(\omega t), \quad (2.2)$$

where h_0 is the amplitude and $\omega\ (=2\pi f)$ is the angular frequency of the oscillating field. The system is in contact with an isothermal heat bath at temperature T.

A square lattice (with periodic boundary condition) of linear size $L\ (=100)$ is considered. The initial condition is that randomly 50% of all spins are up $(+1)$. At any finite temperature T, the dynamics of this system has been studied here by Monte Carlo simulation using Metropolis single spin-flip dynamics [9]. The transition rate is specified as

$$W(s_i \rightarrow -s_i) = \mathrm{Min}[1, \exp(-\Delta H/k_B T)], \quad (2.3)$$

where ΔH is the change in energy due to spin flip ($s_i \rightarrow -s_i$) and k_B is the Boltzmann constant. Any lattice site is chosen randomly and the spin variable (s_i^z) is updated according to the Metropolis probability. L^2 such updates constitute the time unit [Monte Carlo step per spin (MCSS)] here. The magnitude of the field $h(t)$ changes after every MCSS following Eq. (2.2). The instantaneous magnetization (per site) $m(t) = (1/L^2)\Sigma_i s_i^z$ has been calculated.

The time averaged (over the complete cycle of the oscillating magnetic field) magnetization $Q = (1/\tau)\oint m(t)dt$ defines the dynamic order parameter [4]. The frequency is $f = 0.001$ (kept fixed throughout the study). So one complete cycle of the oscillating field takes 1000 MCSS (time period $\tau = 1000$ MCSS). A time series of magnetization $m(t)$ has been generated up to 10^6 MCSS. This time series contains 10^3 (since $\tau = 1000$ MCSS) cycles of the oscillating field. The dynamic order parameter Q has been calculated for each such cycle. So the statistics (distribution of Q) is based on $N_s = 10^3$ different values of Q. The fourth-order cumulant [9] (dynamic order parameter) is defined as

$$U_L = 1.0 - \langle Q^4\rangle / 3\langle Q^2\rangle^2, \quad (2.4)$$

where $\langle Q^n\rangle = \int Q^n P(Q)dQ$ and $P(Q)$ is the normalized $[\int P(Q)dQ = 1]$ distribution of Q. The computational speed recorded is 1.42 million updates per second on an RS6000/43p of an IBM cluster.

III. RESULTS

The statistical distribution $P(Q)$ of dynamic order parameter Q and its temperature dependence have been studied close to the phase boundary to detect the nature of the transition. Figure 1 shows the distributions (at a fixed value of the field amplitude) for three different values of temperature. Below the transition [Fig. 1(a)] the distribution shows only two equivalent peaks centered around ± 1. Close to the transition point [Fig. 1(b)] a third peak centered around zero is developed. As the temperature increases slightly [Fig. 1(c)], the strength of the third peak increases compared to that of the two other (equivalent) peaks. Above the transition [Fig. 1(d)] only one peak is observed centered around zero. This indicates [9] that the transition is first order or discontinuous.

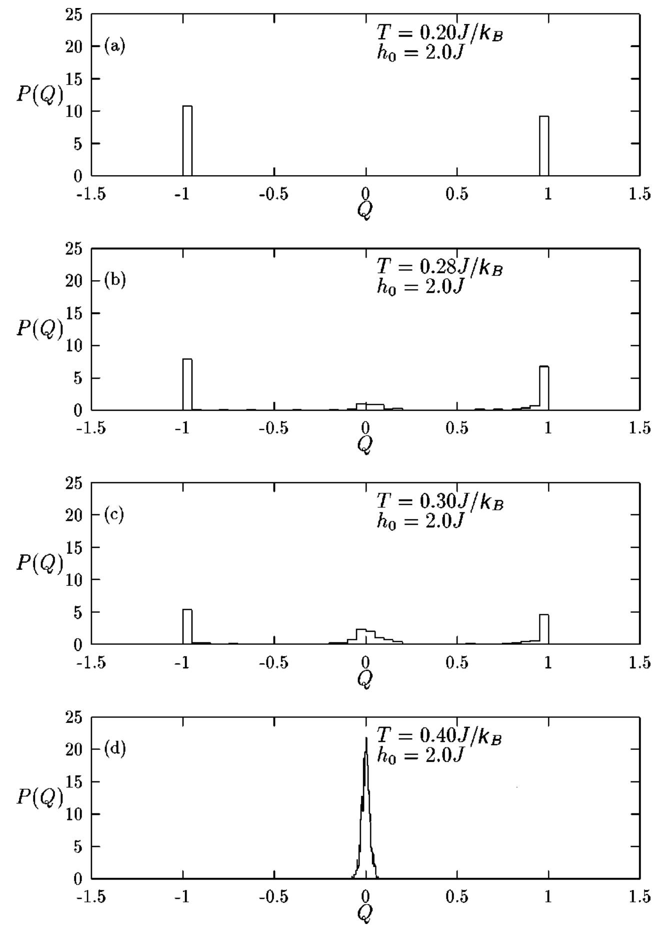

FIG. 1. Histograms of the normalized distributions of the dynamic order parameter Q for different temperatures ($T = 0.20J/k_B, 0.28J/k_B, 0.30J/k_B$, and $0.40J/k_B$) and for the fixed value of the field amplitude h_0. All the figures are plotted in the same scales.

What is the origin of this kind of first-order transition? To get the answer to this question the time variation of the magnetization $m(t)$ is studied (in Fig. 2) for several cycles of the oscillating magnetic field $h(t)$, close to the transition. From Fig. 2 it is clear that sometimes the system stays in the positive well (of the double well form of the free energy) and sometimes it stays in the other well. It is obvious that the best time for the system to switch from one well to the other one is when the value of the field is optimum (''good opportunity'') [3]. So if the system misses one good opportunity (first half period of the oscillating field) to jump to the other well it has to wait for a new chance (another full period of the oscillating field). Consequently, it shows that the residence time (staying time in a particular well) can only be nearly equal to an odd integer multiple of the half period (half of the time period of the oscillating field) [3]. This leads to two consequences.

(a) The distribution of the dynamic order parameter Q would be peaked around three values: (i) $Q \approx 0$ when the system utilizes the good opportunity and goes from one well

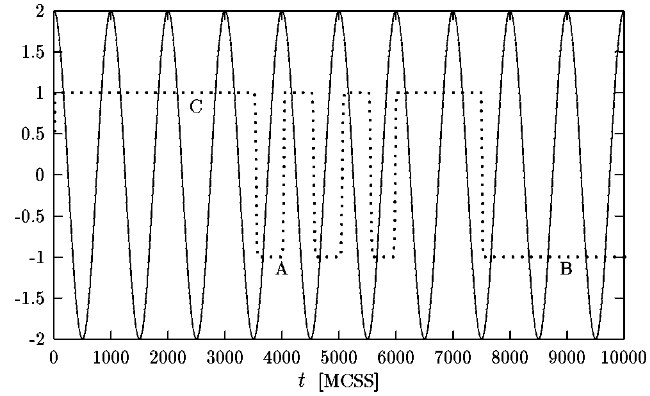

FIG. 2. Time variation of the magnetic field $h(t)$ (solid line) and magnetization $m(t)$ (dotted line) close to the transition ($T = 0.3J/k_B$ and $h_0 = 2.0J$).

to the other (marked A in Fig. 2), (ii) $Q \approx -1$ when the system misses the good opportunity to go from the negative well to the positive well and it stays for one (or more) full period in the negative well (marked B in Fig. 2), and (iii) $Q \approx +1$ when the system misses the good opportunity to go from the positive well to the negative well and spends one (or more) full period in the positive well (marked C in Fig. 2). As a result, the distribution of Q would give three distinct peaks centered at $+1$, -1, and 0.

(b) The other consequence of this kind of time variation of magnetization $m(t)$ is the *stochastic resonance*. This can be detected from the distribution of the residence time (the time the system spends in a particular well). From Fig. 2 it is clear that the distribution P_r of the residence time τ_r will be multiply peaked around the odd integer multiple of the half period [3]. One such distribution is shown in Fig. 3. The distribution shows multiple peaks around the odd integer values (500, 1500, 2500, 3500, 4500, and 5500 MCSS) of the half period ($\tau/2 = 500$ MCSS of the driving fields). The heights of the peaks decrease exponentially (dotted line in Fig. 3) with the peak positions. This is the identifying characteristic of stochastic resonance [3].

The fourth-order cumulant [9] U_L has been plotted against the temperature. In the case of a discontinuous transition, the simultaneous appearance of three peaks (of the distribution of dynamic order parameter) is responsible for a very high value of $\langle Q^4 \rangle$ (compared to the value of $3\langle Q^2 \rangle^2$)

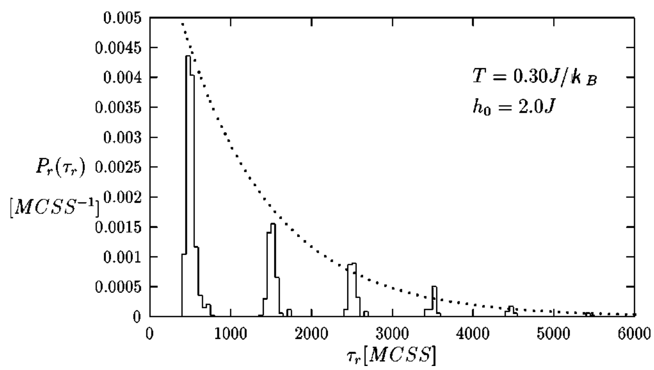

FIG. 3. Histogram of the normalized $[\int P_r(\tau_r) d\tau_r = 1]$ distribution $[P_r(\tau_r)]$ of the residence time (τ_r). The dotted line is the exponential best fit of the envelope of the distribution.

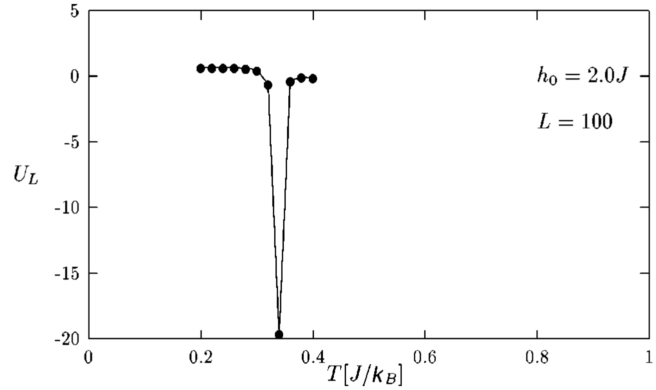

FIG. 4. Temperature (T) variation of the fourth-order Binder cumulant. A deep minimum indicates that the transition is first order and the position of minimum is the transition point.

at the transition point. This will lead to a deep minimum (with a large negative value) of the fourth-order cumulant U_L at the transition point. So the deep minimum corresponds to the first-order transition and the position of minimum is related to the transition point (Fig. 4). From the above observations it is clear that the transition (across the upper part of the dynamic phase boundary) is first order and a manifestation of the stochastic resonance.

Figure 5 shows the distributions of the dynamic order parameter Q for three different values of the temperature. Here the field amplitude h_0 is quite low in comparison to that used in the earlier case (Fig. 1). It shows that, in the ordered region, this gives two (equivalent) peaks [Fig. 5(a)] and as the temperature increases these two peaks come close to each other continuously [Fig. 5(b)] and close to the transition (and also above it) [Fig. 5(c)] only one peak (centered around

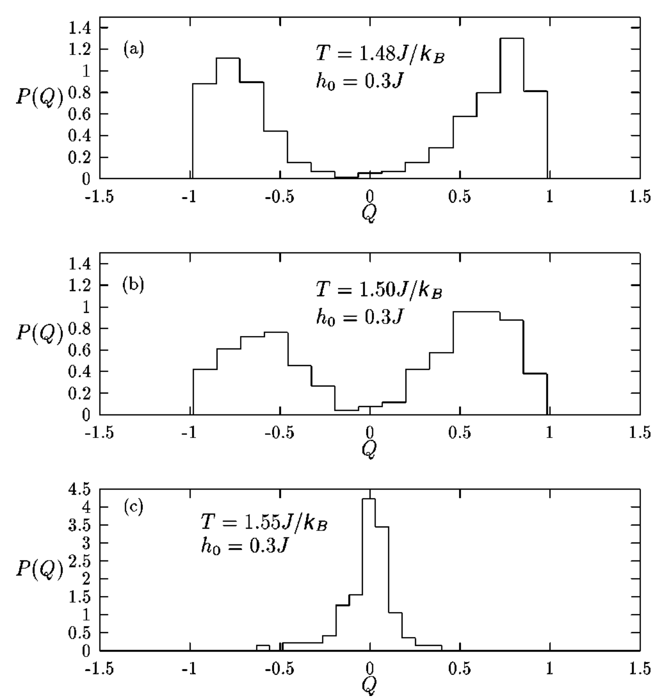

FIG. 5. Normalized distributions of the dynamic order parameter Q (in second order and close to the transition region) for three different temperatures ($T = 1.48J/k_B, 1.50J/k_B$, and $1.55J/k_B$) and fixed field amplitude $h_0 = 0.3J$.

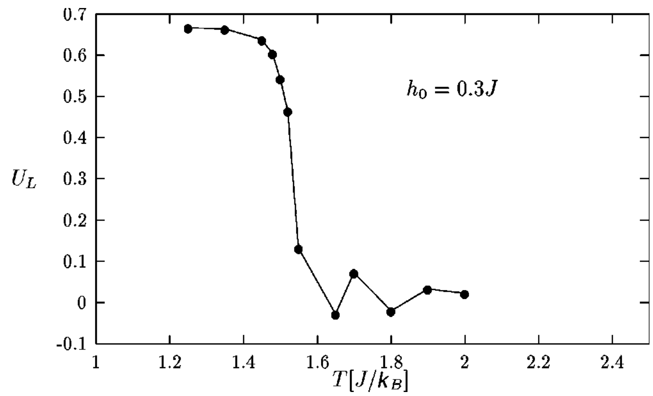

FIG. 6. Temperature (T) variation of the fourth-order cumulant (U_L) for a fixed value of the field amplitude ($h_0 = 0.3J$).

zero) is observed. This feature reveals the continuous or second-order transition [9]. The second-order transition is also characterized by the temperature variation of the fourth-order cumulant U_L [Eq. (2.4)]. Figure 6 shows that U_L continuously decreases from 2/3 to zero revealing the second-order phase transition [9]. It should be mentioned here that the temperature variation of the cumulant and the finite-size study (in the continuous transition region) has been made by Sides *et al.* [7]; here it has been reexamined for completeness. It is important to note that they [7] studied the dynamic transition by varying the frequency (keeping the temperature and field amplitude fixed), whereas the present study has been done by varying the temperature (fixing the frequency and amplitude of the field). However, it is believed that the results are qualitatively invariant under the choice of the tunable parameter.

IV. SUMMARY

The nonequilibrium dynamic phase transition has been studied in the kinetic Ising model in the presence of a time varying (sinusoidal) magnetic field by the Monte Carlo simulation. The nature of the transition is characterized by studying the distribution of the order parameter and the temperature variation of the fourth-order cumulant. For the higher values of the field amplitude the transition observed is discontinuous and for lower values of the field amplitude it is continuous. This indicates that there is a tricritical point [separating the nature (continuous/discontinuous) of the dynamic transition] located on the dynamic phase boundary. These observations support the earlier predictions [2,4] of a TCP on the phase boundary. The residence time distribution shows that the transition is a manifestation of a stochastic resonance. A lengthy computational effort is required to find the precise location of the tricritical point. It would be interesting to know whether or not the TCP can act as a limit of the stochastic resonance (along the first-order line). An extensive investigation is currently under way in this direction and the results will be reported elsewhere.

The detailed finite-size study has been performed by Sides *et al.* [7] and they have not observed any discontinuous transition. They studied the dynamic transition in the very-high-frequency range. For a very high frequency, the tricritical point will shift towards the zero temperature [10] (the region of the first-order transition on the phase boundary will be very short). For this reason Sides *et al.* overlooked the part of the dynamic phase boundary corresponding to the first-order transition. The first-order region of the dynamic phase boundary can be observed clearly in the low-frequency range.

Experimental evidence [11] of the dynamic transition has been found recently. Dynamical symmetry breaking (associated with the dynamic transition) of the hysteresis loop across the transition point has been observed in highly anisotropic (Ising like) and ultrathin Co/Cu(001) ferromagnetic films by the surface magneto-optic Kerr effect. Dynamical symmetry breaking of the hysteresis loop has also been observed [12] in ultrathin Fe/W(110) films. However, the detailed investigation has not yet been made to study the dynamic phase boundary and the nature (continuous/discontinuous) of the transition.

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[10] The tricritical point on the phase boundary appears because of the failure to relax within the time period $(2\pi/\omega)$. This intrinsic relaxation time in the ferromagnetic phase decreases as the temperature decreases and below $T_{TCP}(h_0,\omega)$ the effective relaxation time (τ_{eff}) is less than $2\pi/\omega$. The TCP will be located at the temperature where $\tau_{eff} \approx 2\pi/\omega$. This indicates that TCP will shift towards low temperature as the frequency increases. For a detailed discussion see Ref. [2].
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